"Efficiency of utilization of fast electron beam in atmospheric-pressure electrically excited preionization lasers," Zh. Tekh. Fiz., 45, No. 9, 1982 (1975).
9. F. K. McTaggart, Plasma Chemistry in Electrical Discharges, American Elsevier (1967).
10. L. P. Fominskii, "Electrically excited preionization discharge with electron beam recovery," Zh. Éksp. Teor. Fiz., Pis'ma Red., 21, No. 2, 129 (1975).
11. V. M. Lagunov, A. G. Ponomarenko, and L. P. Fominskii, "The formation of an intense electron beam," Zh. Tekh. Fiz., 42, No. 9, 1947 (1972).
12. O. B. Evdokimov and A. P. Yalovets, "Gas-gap ionization by fast electrons," Zh. Tekh. Fiz., 44, No. 1, 217 (1974).

EVOLUTION OF A ONE-DIMENSIONAL MAGNETOSONIC SOLITON

A. V. Gordeev

UDC 533.951

It is well known that the hydrodynamic equations for cold electrons and ions ($nmc^2 \gg H^2/8\pi \gg nT$) have a solution in the form of a steady-state solitary wave (soliton) [1].

In the present work we investigate the stability of a flow of this sort with respect to electron perturbations of helicon type [2]. The stability of solitons in the Korteweg–de Vries approximation has been investigated by Kadomtsev and Petviashvili [3], who showed that a weak MHD soliton is stable against negative-dispersion perturbations.

We start from the equations of two-fluid hydrodynamics in the quasineutrality approximation $n_e = n_i$:

$$m_e \frac{d\mathbf{v}_e}{dt} = -e\mathbf{E} - \frac{e}{c}[\mathbf{v}_e\mathbf{H}]; \tag{1}$$

$$m_i \frac{d\mathbf{v}_i}{dt} = e\mathbf{E} + \frac{e}{c}[\mathbf{v}_i\mathbf{H}]; \tag{2}$$

$$\operatorname{rot}\mathbf{H} = \frac{4\pi en}{c}(\mathbf{v}_i - \mathbf{v}_e); \tag{3}$$

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}; \tag{4}$$

$$\frac{\partial n}{\partial t} + \operatorname{div}(n\mathbf{v}) = 0. \tag{5}$$

Allowing for electron inertia in the lowest order in the parameter $m_e/m_i \ll 1$, Eqs. (1)-(5) can be used to construct a one-dimensional magnetosonic soliton whose magnetic field $H_z = H_0[1 + h(x - ut)]$ is described by the equation [1]:

$$\pm a\frac{dh}{dx} = \frac{h}{1 - \frac{h + h^2/2}{M^2}}\sqrt{1 - \frac{(2+h)^2}{4M^2}},$$

where

$$a^2 = \frac{m_e c^2}{4\pi e^2 n_0};\ M^2 = \frac{u^2}{H_0^2/4\pi n_0 m_i};\quad u^2 = \frac{(H_0 + H_{\max})^2}{16\,\pi n_0 m_i}.$$

The density n and longitudinal velocity of the soliton v_x are then given by

$$n = \frac{n_0 u}{u - v_x},\ v_x = u\frac{h + h^2/2}{M^2}.$$

We now investigate the stability of the soliton with respect to perturbations, the frequency of which is large: $\omega_{He} > \omega \gg u/a$.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 47-54, July-August, 1977. Original article submitted July 20, 1976.

Equations (1)-(5) give the following estimate for the density variation in such perturbations:

$$\delta n/n_0 \sim (u/a\omega)^2 \, \delta H/H_0. \tag{6}$$

Thus, for high-frequency perturbations, the motion of the ions and the variation of the density can be neglected. The profiles of magnetic field H_z and density n can then be regarded, with the same accuracy, as stationary in a laboratory system of coordinates. This means that the stability of the soliton with respect to high-frequency perturbations can be investigated within the framework of the electron equations:

$$\frac{\partial \mathbf{H}}{\partial t} = \mathrm{rot}\,[\mathbf{vH}] + \frac{m_e c}{e} \mathrm{rot}\, \frac{d\mathbf{v}}{dt}; \tag{7}$$

$$\mathrm{rot}\,\mathbf{H} = -\frac{4\pi e n}{c}\mathbf{v}. \tag{8}$$

We emphasize that the motion of the ions is important for the appearance of instability, as it forms a definite density profile in the soliton. However, this motion proves to be slow with respect to the considered perturbations, and the stability can be investigated assuming the ions have no motion and the density profile is prescribed.

Equations (7) and (8) yield an equation for three-dimensional perturbations which is a generalization of Eq. (8) of [2] with electron inertia for a cold plasma (T = 0) taken into account.

Expanding the perturbations as a Fourier series in time and the coordinates y and z, we obtain

$$\Delta H_x - \frac{n}{H}\left(\frac{H'}{n}\right)' H_x + \frac{\tilde{\omega}\tilde{\tilde{\omega}}}{\omega_H \tilde{\omega}_H} \frac{\omega_p^4}{k_z^2 c^4}\left(H_x - \frac{c^2}{\omega_p^2}\Delta H_x\right) =$$

$$= \frac{\tilde{\omega} n}{k_z^2 \omega_H}\left(\frac{d}{dx}\frac{1}{n}\frac{d}{dx} - \frac{k_\perp^2}{n}\right)\frac{\tilde{\omega}}{\tilde{\omega}_H}\frac{\omega_p^2}{c^2}\left(H_x - \frac{c^2}{\omega_p^2}\Delta H_x\right),$$

where

$$\tilde{\omega} = \omega - k_y \frac{cH'}{4\pi e n}; \quad \tilde{\omega}_H = \omega_H - \left(\frac{cH'}{4\pi e n}\right)';$$

$$\tilde{\tilde{\omega}} = \tilde{\omega} + k_y \left(\frac{c^2}{\omega_p^2}\tilde{\omega}_H\right)';$$

$$\Delta H_x = H_x'' - k_\perp^2 H_x; \quad k_\perp^2 = k_z^2 + k_y^2;$$

and the prime denotes differentiation with respect to x.

Assuming the perturbations to be independent of y(k_y = 0), utilizing the relationship valid for a soliton,

$$a^2 \frac{n_0}{H}\left(\frac{H'}{n}\right)' = 1 - \frac{nH_0}{n_0 H}, \tag{9}$$

and introducing the notation $\nu = n/n_0$, $k^2 = k_z^2 a^2$, $k^2 = k^2 + \nu$, we obtain the following equation for the y component of the vector potential A:

$$A'' - k^2 A - \frac{\nu}{1+h}\left(\frac{h'}{\nu}\right)' A = \gamma_0^2 \frac{\nu}{1+h}\left\{\left[\frac{1}{\nu}\left(\frac{A'' - k^2 A}{\nu}\right)'\right]' - k^2 \frac{A'' - k^2 A}{\nu^2}\right\}, \tag{10}$$

where the dimensionless variable s = x/a; and

$$\gamma_0 = \gamma \frac{4\pi e n_0}{k_z c H_0} a, \quad \gamma = -i\omega.$$

Equation (10), which allows for the inertia of the electrons, is quite complex, and in the general case is not amenable to analytical treatment. Accordingly, we consider it for γ_0^2 1, when the right-hand side of Eq. (10) is small. Along with $\gamma \gg u/a$, this gives the following limitation on γ_0^2:

$$\frac{m_e}{m_i k^2} \ll \gamma_0^2 \ll 1,$$

which is valid for $k_2 \gg m_e/m_i$.

The right-hand side of Eq. (10) contains derivatives of the third and fourth orders; simple discarding of "small" terms is accordingly unjustified, and the equation requires a special analysis [4]. The theory developed in [4] enables us to assert that Eq. (10) with boundary conditions

$$A'(0) = A'''(0) = 0, \quad A(\infty) = A''(\infty) = 0$$

has, for sufficiently small $\gamma_0^2 > 0$, a unique solution $A(s, \gamma_0^2)$ provided that Eq. (10) without the right-hand side and with boundary conditions $A'(0) = A(\infty) = 0$ has a unique solution $A(s)$. It can then be asserted that for $\gamma_0^2 < \varepsilon$, the quantity $|A(s, \gamma_0^2) - A(s)| < \delta(\varepsilon)$.

Let us now find, from physical considerations, that region of small γ_0^2 where the solution $A(s, \gamma_0^2)$ goes over into $A(s)$. To this end we note that the dimensionless potential $U = [\nu/(1 + h)](h'/\nu)'$ in the left-hand side of Eq. (10) varies from $U_0(M) < 0$ at $h_0 = 2(M - 1)$ to zero at $h_1 = (1/2)(\sqrt{8M^2 + 1} - 3)$, then reaches a maximum $U_1(M)$ and falls to zero at $h = 0$. It can be shown by approximating the potential that the permissible values of γ_0^2 are determined from the condition $4U_1\gamma_0^2 < 1$. Utilizing the estimate $U_1 < 1/3$, we find that $\gamma_0^2 \ll 1$ assures coupling of the solutions $A(s, \gamma_0^2)$ and $A(s)$.

The problem of the instability of a magnetosonic soliton for $k_y = 0$ has thus been reduced to the condition for the existence of at least one level for the equation

$$A'' + (E - U)A = 0, \tag{11}$$

where $U = [\nu/(1 + h)](h'/\nu)'$; and the eigenvalue must satisfy the condition $E = k_2 \gg m_e/m_i$.

Equation (11) always has a level provided $\int\limits_{-\infty}^{+\infty} U ds \leqslant 0$.

We introduce the quantity $k(M) = -\int\limits_0^{\infty} U ds$. For $M < 1.3$, the effective potential well in Eq. (11) is shallow and, in accordance with [5], $k = k(M)$. However, the condition $k^2 \gg m_e/m_i$ is then not fulfilled.

With increasing Mach number M, the quantity

$$k(M) = \frac{1}{2M^2} \int\limits_0^{2(M-1)} dh h \frac{3(h^2 + 2h) - 2(M^2 - 1)}{\left(1 - \frac{h + h^2/2}{M^2}\right)^2} \frac{\sqrt{1 - \frac{(2+h)^2}{4M^2}}}{(1+h)^2}$$

varies from $2.17 \cdot 10^{-2}$ at $M = 1.3$ to $1.19 \cdot 10^{-1}$ at $M = 1.5$. In this range of values of M, an eigenvalue k^2 appears that satisfies the condition $1 \gg k^2 \gg m_e/m_i$. Starting from these Mach numbers, the one-dimensional magnetoacoustic soliton proves to be unstable with respect to electron perturbations of helicon type for $k_y = 0$ with a growth rate $\gamma = \gamma_0 k \omega_{He}$ for $\gamma_0^2 \ll 1$. With subsequent increase of $M > 1.5$, the effective potential in Eq. (11) increases and an estimate for the eigenvalue k^2 on the boundary of stability $\gamma_0^2 \ll 1$ can be found from expression (A5) (see Appendix). In order to find γ_0^2 corresponding to k^2 from the range of instability, we must solve the complete fourth-order equation (10).

Let us dwell briefly on the energy balance in the considered instability. Performing an averaging over periodic perturbations in the z direction, we obtain from (7) the following "quasilinear" equation for the main magnetic field H_z for $\gamma_0^2 \ll 1$:

$$\frac{\partial H_z}{\partial t} = \frac{H_0}{2n_0} \frac{\partial}{\partial x} \frac{1}{n} \frac{\partial}{\partial x} \left(\frac{n}{H_z}\right)^2 \frac{\partial}{\partial t} \langle A^2 \rangle, \tag{12}$$

where the angular brackets denote averaging with respect to z.

It can be shown by multiplying (12) by H_z and integrating over all z that the total magnetic-field energy decreases as a result of the growth of this sort of instability and goes over into the energy of the electron motion:

$$\frac{\partial}{\partial t} \int \frac{\mathbf{H}^2}{8\pi} dx = -\frac{\gamma}{4\pi a^2} \int dx \langle A^2 \rangle \frac{n}{n_0} \left(\frac{nH_0}{n_0 H} - 1\right)^2 < 0.$$

However, as the soliton is a "rigid" formation, the reduction of the magnetic field is accompanied by a readjustment of the ion motion. This means that the buildup of fast electron perturbations results in the deceleration of the ions.

If we consider the stability of a soliton with respect to two-dimensional perturbations for $k_z = 0$ and $k_y \neq 0$, it can be shown that, within the framework of purely electron perturbations, the soliton will be stable. It is shown in [2] that an unsteady nonlinear MHD wave is unstable with respect to perturbations with $k_z = 0$. The stability of a soliton for such perturbations is connected with the fulfillment of relationship (9).

In the general case of $k_z \neq 0$ and $k_y \neq 0$, proof of the instability of a magnetosonic soliton involves an analysis of the complete fourth-order equation obtained above, as the right side of the equation is of order $k_y^2/k_z^2 \gtrsim 1$.

Let us consider the question of the existence of a finite steady state reached by a one-dimensional magnetosonic soliton as a result of the growth of electron instability. The growth of electron perturbations leads to a rapid readjustment of the current configuration for a prescribed ion motion. However, for a time on the order of a/u, the plasma flow that occurs must be a solution of the general system of equations (1)-(5).

Let us consider two-dimensional solutions of the form $n(x - ut, z)$ corresponding to motion with velocity u along the x axis. Assuming (as in the one-dimensional case) quasineutrality $n_e = n_i \equiv n$ and taking the velocities of the electrons and ions to be the same along the x and z axes, we introduce the current function $\psi(x, z)$

$$nv_z = \partial\psi/\partial x, \; n(v_x - u) = -\partial\psi/\partial z.$$

As a result we obtain, in the lowest order in the parameter $m_e/m_i \ll 1$, the following system of equations with which to find the profile of a two-dimensional magnetosonic soliton:

$$m_i \left[\frac{\partial v_x}{\partial x}(v_x - u) + v_z \frac{\partial v_x}{\partial z}\right] = -\frac{e}{c} H_z v_y; \tag{13}$$

$$m_i \left[\frac{\partial v_z}{\partial x}(v_x - u) + v_z \frac{\partial v_z}{\partial z}\right] = \frac{e}{c} H_x v_y; \tag{14}$$

$$m_e \left[\frac{\partial v_y}{\partial x}(v_x - u) + v_z \frac{\partial v_y}{\partial z}\right] = -eE_y - \frac{e}{c}(v_z H_x - v_x H_z), \tag{15}$$

where $v_y = (c/4\pi en)\Delta A$, (A is the y component of the vector potential); $H_x = -\partial A/\partial z$; $H_z = \partial A/\partial x$.

Transforming in Eqs. (13)-(15) to the quantities ψ, n, and A, we arrive at the equations:

$$\frac{1}{n^2} \Delta^* \psi \nabla \psi - \nabla \frac{1}{2} \left(\frac{1}{n} \nabla \psi\right)^2 = \frac{\Delta A}{4\pi n m_i} \nabla A; \tag{16}$$

$$\frac{\partial}{\partial x}\left(\frac{c^2}{\omega_p^2}\Delta A - A\right)\frac{\partial \psi}{\partial z} - \frac{\partial}{\partial z}\left(\frac{c^2}{\omega_p^2}\Delta A - A\right)\frac{\partial \psi}{\partial x} = 0, \tag{17}$$

where

$$\Delta^*\psi = n\frac{\partial}{\partial x}\frac{1}{n}\frac{\partial\psi}{\partial x} + n\frac{\partial}{\partial z}\frac{1}{n}\frac{\partial\psi}{\partial z}.$$

We have from Eq. (17):

$$\frac{c^2}{\omega_p^2}\Delta A - A = F(\psi). \tag{18}$$

Inserting this expression into (16) and taking the rot of both sides, we obtain:

$$\frac{1}{n^2}\Delta^*\psi + \frac{F'(\psi)A}{4\pi m_i n_0 a^2} = \Phi(\psi). \tag{19}$$

Finally, inserting the obtained expressions (18) and (19) for A and ψ into (16) and integrating, we obtain the following equation for n:

$$\frac{A^2}{8\pi m_i n_0 a^2} + \frac{AF(\psi)}{4\pi m_i n_0 a^2} + \frac{1}{2}\left(\frac{1}{n}\nabla\psi\right)^2 = \int \Phi(\psi)\,d\psi. \tag{20}$$

Equations (18)-(20) describe the two-dimensional magnetosonic soliton obtained as a result of the splitting up of the front of a one-dimensional magnetosonic soliton along the z axis. We note that in the two-dimensional soliton there is a component of magnetic field H_x along the direction of motion. In accordance with the general theory, Eqs. (18)-(20) depend on two arbitrary functions $F(\psi)$ and $\Phi(\psi)$ [6].

We shall now give some qualitative considerations favoring the existence of a solution of Eqs. (18)-(20) for a particular choice of these functions. Let $F(\psi) = F_0\psi$, $\Phi(\psi) = -\Phi_0\psi^2$, and suppose for the meantime that n is constant. A Fourier expansion for Eqs. (18) and (19) then leads to the following nonlinear equation in **k**-space:

$$\psi_{\mathbf{k}} = \frac{a^2 n_0^2(1+\mathbf{k}^2a^2)\Phi_0}{\mathbf{k}^2a^2(1+\mathbf{k}^2a^2) + \dfrac{n_0^2F_0^2}{4\pi m_i}} \sum_{\mathbf{k}_1} \psi_{\mathbf{k}_1}\psi_{\mathbf{k}-\mathbf{k}_1}. \tag{21}$$

It is shown in [7] that a numerical iteration procedure for (21) converges and gives a localized solution. This same conclusion holds for a sufficiently small variation of the density $\delta n = n(0) - n(\infty)$, the magnitude of δn and the density profile being determined from Eq. (20). In this manner, for values of $M \gtrsim 1.5$, the above-considered instability can lead to a readjustment of a one-dimensional magnetosonic soliton into a two-dimensional. The question of the profile of such a soliton and its stability requires further investigations.

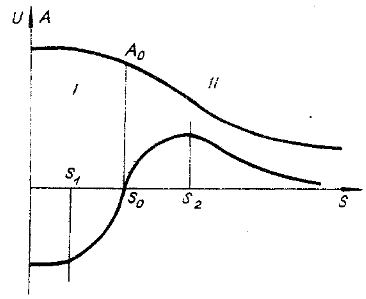

Fig. 1

APPENDIX

1. Consider the equation

$$A'' - (k^2 + U)A = 0, \tag{A1}$$

and let

$$\int_{-\infty}^{+\infty} U ds \leqslant 0,$$

where the potential U has the form shown in Fig. 1.

We introduce test functions in the regions $[0, s_0)$ and (s_0, ∞) which satisfy the boundary conditions $A'(0) = 0$, $A(\infty) = 0$ and which are normalized $A(s_0) \equiv A_0 > 0$.

Then, in region I,

$$A'_I(s_0) = \int_0^{s_0} A(k^2 + U) ds, \quad \text{(A2)}$$

and in region II,

$$A'_{II}(s_0) = -kA_0 - \int_{s_0}^{\infty} AU \exp[-k(s - s_0)] ds. \quad \text{(A3)}$$

We denote $\Delta(k) \equiv A'_{II}(s_0) - A'_I(s_0)$. If, for some $k = k_0$, $\Delta(k_0) = 0$, then the test functions give a solution of Eq. (A1) in the entire range $[0, \infty)$.

We have from (A2) and (A3) that

$$\Delta(k) = -kA_0 - A_0 \int_0^{\infty} U ds - k^2 \int_0^{s_0} A ds + \int_0^{s_0} (A - A_0) |U| ds +$$

$$+ \int_{s_0}^{\infty} ds U \{A_0 - A \exp[-k(s - s_0)]\}. \quad \text{(A4)}$$

For fixed A_0 and $k \to 0$ the integral $\int_0^{s_0} A ds$ is constant, depending only on U. As $A'(s) < 0$ for $s > s_0$, the inequality $A_0 > A > A \exp[-k(s - s_0)]$ is valid, so that $\int_{s_0}^{\infty} ds U \{A_0 - A \exp[-k \times (s - s_0)]\} > 0$. Finally, since $A'(s) < 0$ for $s < s_0$, we have for small k that $\int_0^{s_0} (A - A_0) |U| ds > 0$. Then, for sufficiently small k, $\Delta(k) > -A_0 \int_0^{\infty} U ds > 0$. Let us now consider the case of large k. The integral $\int_0^{s_0} A ds$ remains a positive quantity. For sufficiently large k, $A'(s) > 0$ for $0 < s < s_0$ and $\int_0^{s_0} (A - A_0) |U| ds < 0$, if $k > k_1 \equiv \sqrt{\min |U(s < s_0)|}$. If we now choose k to satisfy the condition $k > \max\left(k_1, \int_0^{s_0} |U| ds\right)$, it follows from (A4) that $\Delta(k) < 0$. Accordingly, for a continuous variation of k, there exists a $k = k_0$ such that $\Delta(k_0) = 0$ and Eq. (A1) has an eigensolution corresponding to $k = k_0$.

2. Let us obtain an estimate for the eigenvalue k^2 of Eq. (A1). To this end we integrate the equation with respect to s from zero to s_3, where $A(s_3) = A_3$ and $k^2 \gg U(s_3)$:

$$-kA_3 = k^2 \int_0^{s_3} A ds + \int_0^{s_3} AU ds.$$

Introducing the notation

$$aA_3 \equiv \int_0^{s_3} A ds, \; bA_3 = -\int_0^{s_3} AU ds,$$

we obtain

$$k = (1/2a)(\sqrt{1-4ba} - 1). \quad \text{(A5)}$$

It follows from (A5) that $k^2 \sim k^2(M)$ for $k(M) \ll 1/4$, and that $k^2 \sim k(M)$ for $k(M) \gg 1/4$. We note that $U_0(M) = -2M(M-1)^2/(2-M)^2(2M-1)$ is not small for all considered Mach numbers M and the well cannot be regarded as shallow.

The author thanks L. I. Rudakov for discussions on the work and V. I. Petviashvili for useful critical remarks.

LITERATURE CITED

1. R. Z. Sagdeev, "Collective processes and shock waves in a rarefied plasma," in: Reviews of Plasma Physics [in Russian], Vol. 4, Gosatomizdat, Moscow (1962), p. 20.
2. A. V. Gordeev and L. I. Rudakov, "Instability of plasma in a strongly nonuniform magnetic field," Zh. Éksp. Teor. Fiz., 55, 2310 (1968).
3. B. B. Kadomtsev and V. I. Petviashvili, "Stability of solitary waves in weakly dispersive media," Dokl. Akad. Nauk SSSR, 192, 753 (1970).
4. W. Wasow, "On the asymptotic solution of boundary-value problems for ordinary differential equations containing a parameter," J. Math. Phys., 23, 173 (1944).
5. L. D. Landau and E. M. Lifshits, Quantum Mechanics [in Russian], Nauka, Moscow (1974), p. 196.
6. L. S. Solov'ev, "Symmetric MHD flows and helical waves in a round cylinder," in: Reviews of Plasma Physics [in Russian], Vol. 3, Gosatomizdat, Moscow (1963), p. 245.
7. V. I. Petviashvili, "On the equation of an unusual soliton," Fiz. Plazmy, 2, No. 3, 469 (1976).

CALCULATION OF TOROIDAL INDUCTIVE ACCUMULATORS WITH A D-CROSS SECTION FROM PARAMETERS OF A DISCHARGE PULSE AND OF THE CHARGING DEVICE

I. A. Ivanov, V. V. Sizov, and V. A. Trukhin UDC 621.039.6

One method of increasing the power of the discharge pulse of an inductive accumulator is based on the use of a scheme with a switch-over: With charging, the sections of the accumulator are connected in series and with discharge, in parallel. In this case, rigid requirements are imposed on the symmetry of the cross section. Such requirements are satisfied by constructions with a toroidal field (another of their advantages is the absence of scattering fields). In [1] it is shown that, for windings on a thin busbar, with a width increasing proportionally to the radius (an s-coil), for a thin conductor of constant width [2] (an l-coil), there exists a profile with which the pressure of the toroidal field is balanced by the voltage of the curved part of the coil. The internal rectilinear section of the coil is subject to compression in the direction of the principal axis and to longitudinal elongation. In a construction with such a profile, with uniform equilization of the radial compression, the action of the bending moments is everywhere completely excluded. The form of the profile, with which the winding does not undergo the action of the bending moments, is called a D-section. A construction with a D-section is optimal with respect to mechanical strength. In [1] it is shown that such a construction is also optimal with respect to energy capacity. Therefore, in comparison with other variants of toroidal constructions in coils with a D-section, the specific parameters are found to be the highest.

1. Method of Calculation

The form and the dimensions are determined by two parameters: the mean radius $r_0 = (r_1 +$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 55-61, July-August, 1977. Original article submitted July 7, 1976.

0021-8944/77/1804-0473$07.50